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Interband multiphoton magneto- and electro-absorption in semiconductor superlattices

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Abstract. Interband optical transitions in a semiconductor superlattice induced by an intense optical wave in the presence of uniform electric and magnetic fields are analysed. Both the oscillating electric field of the optical wave and the uniform magnetic field are directed perpendicular to the heterolayers whilst the uniform electric field is in a direction parallel to the heterolayers. The superlattice potential is modelled by a periodic chain of δ-function-type barriers. Quasi-energetic time-dependent states are used. The explicit dependence of the coefficient of the multiphoton absorption on the frequency and magnitude of the light wave, the superlattice parameters and the magnitudes of the uniform fields is obtained. It is shown that the dynamical Stark effect induced by the strong oscillating electric field leads to a shift of the edge of absorption to shorter wavelengths. The form of magneto-absorption essentially depends upon the parity of the number of absorbed photons and the relationship between the separation of the Landau levels and the total width of the electron and hole minibands. It is found that the main effect of a uniform electric field is to shift the absorption edge to longer wavelengths (the Franz–Keldysh effect).

1. Introduction

At the present time, intensive theoretical and experimental studies of the optical properties of multilayer periodic structures are under way. The most common examples are the heterostructures formed by alternating layers of GaAs and GaAlAs semiconductors, which have similar properties but different forbidden gaps. In such structures, the electrons are under the effect of an additional periodic potential, which influences the electron motion in the direction Oz normal to the heterolayers. This potential consists of a periodic sequence of potential wells separated by barriers and causes changes in the three-dimensional energy band spectrum. The energy spectrum associated with the z-direction of the superlattice is split into an alternating series of allowed and forbidden minibands. This miniband spectrum is superimposed on the two-dimensional band energy spectrum associated with the transverse motion. The overall character of the superlattice energy band spectrum reflects both the localized and extended carrier states of the potential wells due to the tunnelling through the barriers.

Optical experiments, including interband optical absorption, are the methods usually adopted for the investigation of these structures. These techniques are particularly effective in the presence of external electric and magnetic fields. For the case when the electric field

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is along Oz, it is possible to observe experimentally [1] the localization of electrons near the fixed superlattice cell and also the Stark quantization of the electron miniband. This effect has not been observed in bulk semiconductors due to the large width of their energy bands and also because of the smallness of the crystal period. Recently, experiments devoted to a superlattice subjected to both parallel and crossed electric and magnetic fields have been carried out [2–4]. Detailed reviews of such investigations of the electron states and of the optical properties of the superlattices in the presence of external fields have also appeared [5–7].

Many calculations of either a numerical or a variational type of the one-photon absorption of a weak optical wave have been carried out previously. The numerical character of these calculations is a consequence of using real superlattice potentials, which consist of a large number of rectangular wells, separated by barriers with finite width and height. Evidently these potentials cannot be studied by analytical methods. Furthermore, these calculations involve the time-dependent intraband carrier states and ignore the effects of the time-dependent electric field of the optical wave. The calculation of multiphoton transitions based on the independent states is possible in a high order of perturbation theory: for two-photon processes, second-order perturbation theory is required, for three-photon processes, third-order theory is required, and so on. These calculations rapidly become very cumbersome and thus only one- or two-photon absorption can be considered in practice.

An analytical approach to the calculation of the interband optical transitions in a semiconductor superlattice has been developed recently [8, 9] in which the superlattice potential barriers are modelled by barriers that are δ-type functions. Explicit analytical expressions for the time-independent carrier states and for the coefficient of one-photon absorption in the presence of electric [8] and magnetic fields [9] were obtained. The results are in good agreement with both numerical calculations and experimental data [1, 5].

In this paper, we investigate two items connected with the theory of optical absorption in the presence of external fields. Time-dependent intraband states induced by an intense optical wave, and interband multiphoton magneto- and electro-absorption, have not been considered previously. Thus an analytical theory based on the time-dependent quasi-energetic carrier states in the presence of external fields is developed here. The external field is a superposition of the time-dependent electric field of the optical wave and the uniform electric and magnetic fields. The oscillating electric and uniform magnetic fields are both along the Oz direction and the uniform electric field is parallel to the layers in the x–y plane. For this direction of the oscillating electric field, the interaction between it and the superlattice potential is greatest. Also, for this direction of the magnetic field, the effects of the magnetic field and the oscillating electric field are separated. In contrast to other directions of the magnetic field, it is possible to carry out the calculations in an analytical form. When the uniform electric field is parallel to the oscillating electric field, the localization of the carriers require special consideration. The limiting form of the Kronig–Penney potential is used to model the superlattice potential which is represented by a periodic chain of δ-function-type barriers. The general expression for the coefficient of the interband multiphoton absorption in the superlattice is derived. The explicit dependence of the coefficient of absorption on the frequency and magnitude of the optical wave, the superlattice parameters and the magnitudes of the uniform fields is obtained. The multiphoton magneto- and electro-absorption is also discussed.

In the analysis to be described below, it is shown that the allowed minibands are narrowed whilst the forbidden bands expand due to the dynamical Stark effect induced by the intense oscillating electric field. This leads to a shift of the edge of the magneto-absorption to shorter wavelengths. The magneto-absorption spectrum associated with transitions between

the ground hole miniband and the ground electron miniband gives a series of equidistant continuous absorption bands. The energy separations between the bands are equal to the separations of the Landau levels of the electron and hole bands. The form of absorption depends explicitly upon the parity of the number of absorbed photons. An absorption band involving an odd number of photons gives singularities near the band edges whilst a minimum occurs in the centre of the band. In contrast, the absorption tends to zero near the band edges for an even number of photons. The intensities of the band are equal; their overall shape depends upon the separation of the Landau levels and the widths of the electron and hole minibands. This relationship defines whether these bands overlap or are separated in a spectrum.

It is also found that the effect of a uniform electric field is to shift the absorption edge to longer wavelengths (the Franz–Keldysh effect). In the general case, the intensities of the magneto- and electro-absorptions depend upon the number of absorbed photons and are found to decrease rapidly as this number increases.

2. Quasi-energetic electron states in the superlattice

Let us consider an electron of charge e in a semiconductor superlattice with a large number N' of periods a in the presence of an oscillating electric field $\eta F_0 \cos \omega t$, of angular frequency ω, magnitude F_0, polarization (unit) vector η, an external uniform electric field $\boldsymbol{E}$ and a uniform magnetic field $\boldsymbol{B}$. The total electric field is then given by

$$\boldsymbol{F}(t) = \boldsymbol{E} + \eta F_0 \cos \omega t. \tag{2.1}$$

Assuming that the external fields, including the superlattice potential, satisfy the effective mass approximation, the equation for the envelope wavefunction Ψ describing a particle at a position $\boldsymbol{r}$ in a simple band with effective mass m is given by

$$\{(1/2m)(-i\hbar\nabla - (e/2)[\boldsymbol{B} \cdot \boldsymbol{r}])^2 + V(z) - e\boldsymbol{F}(t) \cdot \boldsymbol{r}\}\Psi(\boldsymbol{r}, t) = i\hbar\partial\Psi(\boldsymbol{r}, t)/\partial t \tag{2.2}$$

where

$$V(z) = \alpha_0 \sum_s \delta(z - as) \qquad V(z) = V(z + a) \qquad \alpha_0 > 0 \qquad s \text{ integer} \tag{2.3}$$

is the periodic superlattice potential formed by the δ-function-type barriers of power α_0.

Transitions between the bands separated by a wide energy gap E_g occur via the operator

$$P_{eh}(t) = P_0 \cos \omega t \qquad P_0 = i\hbar e F_0 (\eta \cdot p_{eh})/m_0 E_g \tag{2.4}$$

for allowed electric dipole transitions [10] where p_{eh} is the matrix element of momentum between the amplitudes of the Bloch functions of the electron and hole bands and m_0 is the mass of a free electron. For uniform fields in the directions $\boldsymbol{B} \| Oz$ and $\boldsymbol{E} \| -Ox$, the solution to equation (2.2) is

$$\Psi(\boldsymbol{r}, t) = e^{-iE_\perp t/\hbar} \Phi_\perp(\rho)\phi(z, t) \tag{2.5}$$

where $\Phi_\perp(\rho)$ is the transverse function of a particle with energy $E_\perp$ in the uniform fields $\boldsymbol{B}$ and $\boldsymbol{E}$, and $\phi(z, t)$ obeys the equation

$$-(\hbar^2/2m)\partial^2\phi/\partial z^2 + [V(z) - eF_0 z \cos \omega t]\phi(z, t) = i\hbar\partial\phi/\partial t. \tag{2.6}$$

The solutions to this equation with $F_0 = 0$ are

$$\phi_n^{(0)}(z, t, k) = e^{-i\varepsilon_n(k)t/\hbar}\psi_n(z, k) \tag{2.7}$$

where

$$\varepsilon_n(k) = \varepsilon_n(k + 2\pi/a) \qquad \psi_n(z, k) = \psi_n(z, k + 2\pi/a) \tag{2.8}$$

giving allowed bands labelled sequentially by the index n. The functions $\psi_n(z, k)$ are the Bloch functions of the superlattice where k is the average momentum of a particle in the superlattice. An explicit form for weak barrier penetration for which

$$\lambda = \hbar^2/2ma\alpha_0 \ll 1$$

can be found [8] where λ is the reciprocal dimensionless barrier power.

The expression for the energy spectrum is

$$\varepsilon_n(k) = b_n + \tfrac{1}{2}\Delta_n(1 - \cos ka) \tag{2.9}$$

where

$$b_n = (\hbar^2/2m)(n\pi/a)^2[(1 - 2\lambda)^2 + 4(-1)^n\lambda]$$

and

$$\Delta_n = 8(-1)^{n+1}(\hbar^2/2m)(n\pi/a)^2\lambda \qquad n = 1, 2, 3, \ldots.$$

The lower boundary of each miniband is $\varepsilon_n(0) = b_n$, and the corresponding width is Δ_n. We shall consider the ground minibands only, so the $n = 1$ label will henceforth be dropped.

If we take into account the relation [8]

$$\partial\psi/\partial k = iz\psi(1 + O(\lambda)) \qquad z \simeq as \tag{2.10}$$

where $O(\lambda)$ is a function of order λ, the solution to equation (2.6) is the function

$$\phi(z, t, k) = \psi[z, q(t, k)] \exp\left\{-\frac{i}{\hbar}\int_0^t \varepsilon[q(\tau, k)]\,d\tau\right\} \tag{2.11}$$

where

$$q(\tau, k) = k + (eF_0/\hbar\omega)\sin\omega\tau.$$

It is easy to see that this function is a quasi-energetic solution satisfying the condition [11]

$$\phi(z, t + T, k) = \exp\{-(i/\hbar)\mathcal{E}T\}\phi(z, t, k) \qquad T = 2\pi/\omega \tag{2.12}$$

with quasi-energy

$$\mathcal{E}(k) = \frac{1}{T}\int_0^T \varepsilon[q(\tau, k)]\,d\tau. \tag{2.13}$$

This quasi-energetic solution can be represented in the form

$$\phi(z,t,k) = \mathrm{e}^{-\mathrm{i}\mathcal{E}t/\hbar} f(z,t,k) \qquad f(z,t,k) = f(z,t+T,k) \tag{2.14}$$

such that

$$f(z,t,k) = \mathrm{e}^{\mathrm{i}\mathcal{E}t/\hbar}\psi[z,q(t,k)]\exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \varepsilon[q(\tau,k)]\,\mathrm{d}\tau\right\}.$$

The functions are then expanded in a Fourier series to take into account the periodicity of the functions (2.8) in the form

$$\psi(z,q) = \frac{1}{\sqrt{N'}}\sum_l Q_l(z)\mathrm{e}^{\mathrm{i}qal} \tag{2.15}$$

where l is summed over the superlattice indexes and where

$$Q_l(z) = \frac{a}{2\pi}\sqrt{N'}\int_{-\pi/a}^{+\pi/a}\psi(z,q)\mathrm{e}^{-\mathrm{i}qal}\,\mathrm{d}q \tag{2.16}$$

and the particle Wannier functions such that

$$\langle Q_{l'}|Q_l\rangle = \delta_{ll'}. \tag{2.17}$$

It follows from the explicit form of the Wannier functions and the condition $\lambda \ll 1$ that these functions are not equal to zero in the superlattice cells with indices $s = l, l+1, l-1$ with $Q_l \sim \lambda^{|s-l|}$. The expansion (2.15) and the formulae (2.5), (2.13) and (2.14) define the quasi-energetic time-dependent state of a carrier in the miniband.

3. The coefficient of the interband multiphoton absorption

Let us consider the interband absorption as a transition of an electron–hole pair from the ground state to an excited state to give an electron at position r_e in the conduction band and a hole at position r_h in the valence band. The ground state is then described by the function [12]

$$\Psi_0(r_\mathrm{e}, r_\mathrm{h}) = \delta(r_\mathrm{e} - r_\mathrm{h}) \tag{3.1}$$

and the excited state by the function

$$\Psi(r_\mathrm{e}, r_\mathrm{h}, t) = \Psi_\mathrm{e}(r_\mathrm{e}, t)\Psi_\mathrm{h}(r_\mathrm{h}, t). \tag{3.2}$$

The electron function $\Psi_\mathrm{e}(r_\mathrm{e}, t)$ is defined by equation (2.5) with the addition of subscripts 'e' to all parameters (r, m, b, Δ, k, λ, $\mathcal{E}_\perp$ and $\mathcal{E}$). Similarly, the hole function $\Psi_\mathrm{h}(r_\mathrm{h}, t)$ is obtained by the addition of subscripts 'h' and with $k_\mathrm{e} \to -k_\mathrm{h}$, $t \to -t$, $e \to -e$ and taking complex conjugates. In addition, in the expansion (2.15), the electron Wannier function $Q_l(z_\mathrm{e})$ should be replaced by the hole function $\bar{Q}^*_{l'}(z_\mathrm{h})$ with $\bar{Q}_l = Q_{-l}$.

The coefficient of the interband dipole transition under the oscillating electric field effect is defined by the matrix element of the electric dipole operator (2.4)

$$S(t) = \frac{1}{\mathrm{i}\hbar}\int_0^t \delta(r_\mathrm{e} - r_\mathrm{h})P_\mathrm{eh}(\tau)\Psi^*(r_\mathrm{e}, r_\mathrm{h}, \tau)\,\mathrm{d}r_\mathrm{e}\,\mathrm{d}r_\mathrm{h}\,\mathrm{d}\tau. \tag{3.3}$$

The relations between the transition rate W and the coefficient of absorption α are

$$\alpha = \frac{n_0 \hbar \omega W}{cu\Omega} \qquad W = \frac{1}{t}\sum_{e,h} |S(t)|^2 \tag{3.4}$$

where $n_0(\omega)$ is the refractive index, c is the speed of light, $u(= \varepsilon_0 n_0^2 F_0^2)$ is the optical energy density, $\Omega = L_x L_y N' a$ is the volume of the crystal, $\sum_{e,h}$ is a sum over all band states, and L_x, L_y are transverse dimensions of the crystal.

Substituting the operator P_{eh} and the function $\Psi(r_e, r_h, t)$ into (3.3) with (2.5), (2.4) and (3.2), the result for $S(t)$ is

$$S(t) = \frac{P_0}{i\hbar}\int_0^t \cos\omega\tau M(\tau) \exp\left\{\frac{i}{\hbar}(\mathcal{E}_g + \mathcal{E}_{\perp e} + \mathcal{E}_{\perp h} + \mathcal{E}_e + \mathcal{E}_h)\tau\right\} d\tau \tag{3.5}$$

where

$$M(\tau) = \int \delta(r_e - r_h)\phi^*_{\perp e}(\rho_e)\phi^*_{\perp h}(\rho_h) f_e^*(z_e, \tau) f_h^*(z_h, \tau)\, dr_e\, dr_h \tag{3.6}$$

such that $M(\tau + 2\pi/\omega) = M(\tau)$.

Using the periodicity of $M(\tau)$, we expand it in a Fourier series such that

$$M(\tau)\cos\omega\tau = \sum_{\ell=-\infty}^{+\infty} e^{-i\ell\omega\tau} A_\ell(\omega) \tag{3.7}$$

where

$$A_\ell(\omega) = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{+\pi/\omega} e^{i\ell\omega t}\cos\omega t M(t)\, dt \tag{3.8}$$

is the inverse relation. This in turn is substituted into (3.5) and (3.4), so that the general form for the coefficient of absorption α becomes

$$\begin{aligned} &\alpha = \sum_\ell \alpha_\ell \\ &\alpha_\ell(\omega) = \frac{2\pi\omega\hbar^2 e^2 |\eta \cdot p_{eh}|^2}{\varepsilon_0 c \Omega m_0^2 \mathcal{E}_g^2 n_0}\sum_{e,h} |A_\ell(\omega)|^2 \delta(\ell\hbar\omega - \mathcal{E}_g - \mathcal{E}_{\perp e} - \mathcal{E}_{\perp h} - \mathcal{E}_e - \mathcal{E}_h) \end{aligned} \tag{3.9}$$

where α_ℓ is the coefficient of ℓ-photon interband absorption.

If we neglect the time dependence of the function f (equation (2.4)) induced by the variable electric field, it follows that, from equations (3.6), (3.8) and (3.9), only the calculation of the one-photon absorption is possible.

4. The calculation of the Fourier coefficients

To find the longitudinal wavefunction (2.14) and quasi-energy (2.11), (2.13) in an explicit form, we take into account that, for real superlattices and oscillating field F_0, the parameter

$$\beta = eF_0 a/\hbar\omega \ll 1. \tag{4.1}$$

If $\hbar\omega \simeq \mathcal{E}_g \simeq 1$ eV, $a = 50$ Å and $F_0 = 5 \times 10^7$ V m^{-1}, then $\beta = 0.24$. It follows from (2.14), (2.13), (2.11) and (2.9) that

$$f(z, t, k) = \psi[z, q(t, k)]v(t, k) \tag{4.2}$$

where

$$v(t, k) = \exp\{-\tfrac{1}{2}i\zeta[\beta \sin ka(1 - \cos\omega t) - \tfrac{1}{8}\beta^2 \cos ka \sin 2\omega t]\} \qquad \zeta = \Delta/\hbar\omega \tag{4.3}$$

(where Δ and b are taken for the particle under consideration) and

$$\mathcal{E}(k) = b + \tfrac{1}{2}\Delta[1 - \cos ka(1 - \tfrac{1}{4}\beta^2)]. \tag{4.4}$$

Thus an intense oscillating electric field moves the minimum of the allowed electron miniband $\mathcal{E}(0)$ to the high-frequency region by $\frac{1}{8}\Delta\beta^2$. Its maximum $\mathcal{E}(\pi/a)$ reduces to the same value. As a result, the allowed minibands are narrowed due to a dynamic Stark effect such that

$$\Delta(F_0) = \Delta(0)(1 - \tfrac{1}{4}\beta^2). \tag{4.5}$$

The expansion (2.15) of the Bloch function is now used by substituting the combined expression into (3.6). A transformation to the relative coordinates r and the coordinate of the centre of mass R is made using the relations

$$r = r_e - r_h \qquad R = (m_e r_e + m_h r_h)/(m_e + m_h) \tag{4.6}$$

and writing

$$\Phi_{\perp e}(\rho_e)\Phi_{\perp h}(\rho_h) = [e^{iK_\perp \cdot R_\perp}/(L_x L_y)^{1/2}]\phi_\perp(\rho) \tag{4.7}$$

where $K_\perp$ is the total transverse momentum of the electron–hole pair and $\Phi_\perp(r)$ is a function of the relative transverse motion. In the dipole approximation $K_\perp \simeq 0$ so that we have

$$M(\tau) = \sqrt{L_x L_y}\Phi_\perp(0)\frac{1}{N'}\sum_{l,l'}\langle Q_l^*|Q_{l'}\rangle e^{-ia(lk_e + l'k_h)}e^{i(l'-l)\beta \sin\omega\tau}v_e^*(\tau, k_e)v_h^*(\tau, k_h). \tag{4.8}$$

The overlap integrals of the Wannier functions of the electron and hole are given in explicit form in [8]. They are typically $\langle Q_l^*|Q_{l'}\rangle \sim \lambda^{|l'-l|}$. For $\lambda \ll 1$ and using $\langle Q_l^*|Q_{l'}\rangle = \delta_{ll'}$, we have

$$\sum_l \exp[-ia(k_e + k_h)l] = N'\delta_{k_e - k_h} \tag{4.9}$$

and an explicit form of the functions v (4.3) for $k_e = -k_h = k$, by substituting the formulae (4.8) into equation (3.8). In the last equation, we define $\sin\omega t = u$, $\sin\omega\tau = v$ and find that

$$A_\ell(\omega, k) = \sqrt{L_x L_y}\Phi_\perp(0)B_\ell(\omega, k) \tag{4.10}$$

where

$$B_\ell(\omega,k)=\frac{1}{2\pi}\oint \mathrm{d}u\,\exp\left\{ig\int_0^u\frac{\mathrm{d}v}{\sqrt{1-v^2}}\left(1+\frac{2\xi}{\gamma}v+\frac{v^2}{\gamma^2}\right)\right\} \tag{4.11}$$

with

$$g=\ell-\tfrac{1}{8}\zeta_{\mathrm{eh}}\beta^2\cos ka \qquad \zeta_{\mathrm{eh}}=\zeta_{\mathrm{e}}+\zeta_{\mathrm{h}}$$

and

$$2\xi(k)=(\zeta_{\mathrm{eh}}/g)^{1/2}\sin ka/(\cos ka)^{1/2} \qquad 1/\gamma^2(k)=(\zeta_{\mathrm{eh}}/4g)\beta^2\cos ka.$$

In the plane of the complex variable u, the line of integration spans the cut from $u=-1$ to $u=+1$. The integral in (4.11) has been calculated by the saddle-point method [10, 13], where $g\simeq\ell$ is a large parameter.

For wide-gap semiconductors (with $\mathcal{E}_{\mathrm{g}}\geqslant\hbar\omega$) the total width of the minibands $\Delta_{\mathrm{e}}+\Delta_{\mathrm{h}}$ is small in comparison with $\mathcal{E}_{\mathrm{g}}$. Therefore $\zeta_{\mathrm{eh}}\ll 1$, and $\gamma^2\gg 1$. In this case, the coefficient $B_\ell(\omega,k)$ simplifies to

$$B_\ell(\omega,k)=(1/2\sqrt{\ell\pi})\mathrm{e}^{-\mathrm{i}(\pi/2)(\ell-1)+\ell/2}(1/2\gamma)^{\ell-1}\sin(-2\ell\xi+\ell\pi/2). \tag{4.12}$$

Note that the results obtained above have a common character; they have no relation to the standard transverse states.

5. Interband multiphoton magneto-absorption

Let us consider the interband optical absorption in the presence of an external uniform magnetic field $\boldsymbol{B}\|\mathrm{Oz}$ and $\boldsymbol{E}=0$. The wavefunctions $\Phi_\perp(\rho)$ and energies $\mathcal{E}_{\perp\mathrm{e,h}}$ (Landau levels) are well known [8]:

$$\Phi_\perp(0)=1/\sqrt{2\pi}a_{\mathrm{H}} \qquad \mathcal{E}_{\perp N}=\mathcal{E}_{\perp\mathrm{e}}+\mathcal{E}_{\perp\mathrm{h}}=(\hbar eB/\mu)(N+\tfrac{1}{2}) \tag{5.1}$$

where

$$a_{\mathrm{H}}=(\hbar/eB)^{1/2} \qquad \mu^{-1}=m_{\mathrm{e}}^{-1}+m_{\mathrm{h}}^{-1} \qquad N=0,1,2,\ldots.$$

We substitute $\Phi_\perp(0)$ and $\mathcal{E}_{\perp N}$ from (5.1), and the coefficient B_ℓ (4.12) into (4.10) and (3.9). In equation (4.12), we assume $\xi=0$ for odd ℓ. For even ℓ we should keep the dependence $B_\ell(\xi)$. The coefficient of absorption α_ℓ (3.9) then becomes

$$\alpha_\ell^{\mathrm{u,g}}(\omega)=A'(e^\ell/4\ell\pi)(\zeta_{\mathrm{eh}}\beta^2/16\ell)^{\ell-1}(\ell\zeta_{\mathrm{eh}})^p\Lambda_\ell^{\mathrm{u,g}}(\omega) \tag{5.2}$$

where

$$A'=\frac{\omega e^2|\eta\cdot p_{\mathrm{eh}}|^2 l}{\pi\varepsilon_0 acm_0^2\mathcal{E}_{\mathrm{g}}^2 n_0} \qquad p=\begin{cases}0 & \text{odd } \ell\\ 1 & \text{even } \ell\end{cases}$$

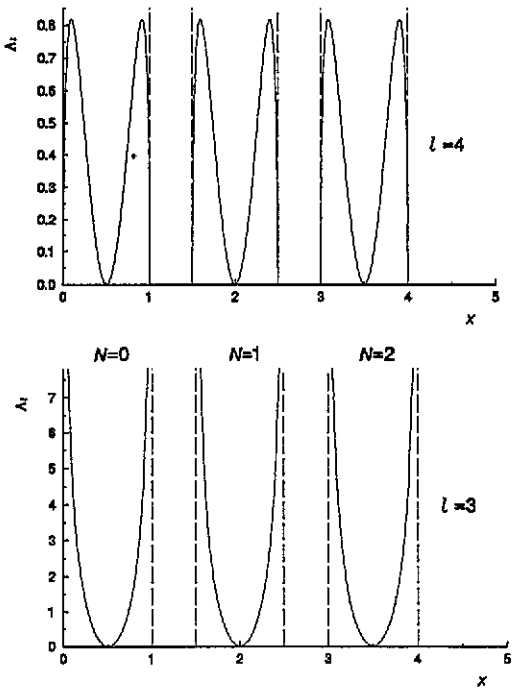

Figure 1. A plot of the superlattice magneto-absorption $\Lambda_l^{g,g}$ as a function of $X = [\omega - \omega_-(0)]/[\omega_+(N) - \omega_-(N)]$ for $\hbar eB/\mu(\Delta_e + \Delta_h) = \frac{3}{2}$ and $l = 3$ and 4.

and the absorption connected with the Landau level N is given by

$$\Lambda_\ell^{u,g}(\omega) = 2\sum_N \frac{eB}{\mu\ell[\omega_+(N) - \omega_-(N)]} \left\{\frac{2[\omega_+(N) - \omega][\omega - \omega_-(N)]}{[\omega_+(N) - \omega_-(N)]^2}\right\}^{\pm 1/2} \times \left[\frac{2\omega - \omega_+(N) - \omega_-(N)}{\omega_+(N) - \omega_-(N)}\right]^{\ell-1-p} \tag{5.3}$$

where

$$\ell\hbar\omega_-(N) = \mathcal{E}_g + (b_e + b_h) + (\Delta_e + \Delta_h)\beta^2/8 + (\hbar eB/\mu)(N + \tfrac{1}{2}) \tag{5.4}$$

and

$$\ell\hbar\omega_+(N) = \mathcal{E}_g + (b_e + b_h) + (\Delta_e + \Delta_h)(1 - \tfrac{1}{8}\beta^2) + (\hbar eB/\mu)(N + \tfrac{1}{2}) \tag{5.5}$$

with the upper and lower sign in equation (5.3) corresponding to even (g) and odd (u)ℓ respectively.

The form of the superlattice magneto-absorption spectrum is depicted in figure 1 for $\ell = 3$ and 4 as a plot of $\Lambda_\ell^{u,g}$ as a function of the dimensionless measure of frequency

$$X = [\omega - \omega_-(0)]/[\omega_+(N) - \omega_-(N)]$$

and for the ratio

$$\hbar eB/\mu(\Delta_e + \Delta_h) = \tfrac{3}{2}.$$

The optical absorption gives a series of continuous absorption bands connected with the Landau levels $\mathcal{E}_{\perp N}$, $N = 0, 1, 2, \ldots$. The absorption is in the region $\omega_-(N) \leqslant \omega \leqslant \omega_+(N)$. There is no absorption outside the band. The distance between the neighbouring bands is equal to the separation and the distance $\Delta\mathcal{E}_\perp = \hbar eB/\mu$ between the Landau levels. The width of the absorption band is equal to the total width of the electron and hole minibands

$$\ell\hbar[\omega_+(N) - \omega_-(N)] = (\Delta_e + \Delta_h)(1 - \tfrac{1}{4}\beta^2). \tag{5.6}$$

If the distance between the Landau levels $\hbar eB/\mu$ exceeds the width of the band $\Delta_e + \Delta_h$, the absorption bands are separated by a gap in the spectrum. In the opposite case there is a superposition of the bands.

It follows from (5.3) that the intensities of all bands are equal, with the intensity depending on the ratio

$$\hbar eB/\mu(\Delta_e + \Delta_h).$$

The shape of the absorption band depends on the parity of the number of photons ℓ. For odd ℓ, the absorption has reciprocal root singularities $[\omega_+(N) - \omega]^{-1/2}[\omega - \omega_-(N)]^{-1/2}$ near the boundaries of the band. In the centre of the band, $\omega = \frac{1}{2}[\omega_+(N) + \omega_-(N)]$ and the absorption is equal to zero except for the one-photon absorption for which $\ell = 1$, $m = 0$. For even ℓ, the absorption tends to zero near the boundaries having the form $[\omega_+(N) - \omega]^{1/2}[\omega - \omega_-(N)]^{1/2}$. In the centre of the band, the absorption is equal to zero except for the two-photon absorption $\ell = 2$, $m = 0$. Notice that the intensity of the multiphoton absorption decreases with the number of photons such that $\alpha_\ell \sim (\zeta\beta^2)^{\ell-1}$ and $\zeta\beta^2 \ll 1$. To observe the transitions with $\ell = 2, 3$ in the GaAs/GaAlAs structure, the optical wave should have an intensity of about 10^5 MW m^{-2}. In particular, the total width of the electron and hole minibands $(\Delta_e + \Delta_h)$ could be determined from the experimental spectrum. In accordance with (5.6), this important parameter is determined by the distance between the boundary frequencies $[\omega_+(N) - \omega_-(N)]$, which corresponds to the singularities in the odd-photon magneto-absorption spectrum (5.3). Indeed, such singularities in the one-photon magneto-absorption spectrum have been used by Maan *et al* [14] to measure the width of the ground conduction miniband in InAs/GaSb superlattices.

We will obtain now some estimates using the parameters of GaAs in a GaAs/$Ga_{1-x}Al_xAs$ heterostructure with $x = 0.35$, $m_e = 0.065m_0$, $m_{hh} = 0.55m_0$, $m_{\ell h} = 0.09m_0$ and $\mathcal{E}_g = 1.53$ eV. We assume further that the superlattice has a typical period of 50 Å and that $\lambda_e = 0.05$. We note that the selected quantity λ_e corresponds to the electron miniband width $\Delta_e = 0.0926$ eV, which is very close to the value of 0.0950 for Δ_e given in [5] and obtained by numerical calculation using a model of rectangular wells of width 40 Å separated by barriers of width 15 Å. Other parameters are $\lambda_{hh} = 0.006$, $\lambda_{\ell h} = 0.036$, $\Delta_{hh} = 0.0013$ eV and $\Delta_{\ell h} = 0.048$ eV. The total width of the minibands is given by $\Delta_e + \Delta_{\ell h} = 0.14$ eV for light holes and is $\Delta_e + \Delta_{hh} = 0.094$ eV for heavy holes.

In a magnetic field $B = 10$ T, $\Delta\mathcal{E}_{\perp,\ell h} = 0.030$ eV for the light hole and $\Delta\mathcal{E}_{\perp,hh} = 0.015$ eV for the heavy hole. The distance is less than the width of the absorption band and therefore there is a superposition of the absorption bands in the spectrum. The separation of the bands can be realized in structures in which there is anisotropy in the energy bands and also a strong magnetic field. These bands are characterized by small transverse masses and large longitudinal masses. This situation arises in the GaInAs/GaAlAs superlattice. When this structure is subjected to a magnetic field $B = 20$ T, the total width of the electron and heavy-hole minibands is less than the distance between the Landau levels. In this case, the

shape of the experimental one-photon magneto-absorption spectrum [15] is in very close agreement with the calculated shape given in (5.3) and shown in figure 1.

It follows from equation (5.3) that, in the limiting case of $\omega-\omega_-(N) \ll \omega_+(N)-\omega_-(N)$, we have

$$\Lambda_\ell^{u,g}(\omega) \sim \sum_N (\omega - \omega_-(N))^{\pm 1/2}.$$

This coincides with the results valid for bulk semiconductors [10, 13].

6. Interband multiphoton electro-absorption

If the superlattice is under the effect of a uniform electric field $E \| -Ox$ and $B = 0$, the transverse states with energy $\mathcal{E}_\perp$ are described by the function

$$\begin{aligned} &\Phi_\perp(\rho) = [e^{ik_y y}/(2\pi)^{1/2}][(eE)^{1/2}/\mathcal{E}_0]\mathrm{Ai}[-(eE/\mathcal{E}_0)(x + \mathcal{E}_x/eE)] \\ &\mathcal{E}_0 = (\hbar^2 e^2 E^2/2\mu)^{1/3} \end{aligned} \tag{6.1}$$

where

$$\mathcal{E}_\perp = \mathcal{E}_{\perp e} + \mathcal{E}_{\perp h} = \mathcal{E}_x + \hbar^2 k_y^2/2\mu$$

and where $\mathrm{Ai}[u]$ is the Airy function [16]. The normalization of the function (6.1) is defined by the relation

$$\int \mathrm{d}\rho\, \phi_\perp(\rho, \mathcal{E}_x, k_y)\Phi_\perp^*(\rho, \mathcal{E}_x', k_y') = \delta(k_y - k_y')\delta(\mathcal{E}_x - \mathcal{E}_x').$$

Using the expressions (4.12) for B_ℓ and (4.10) for A_ℓ together with the coefficient of absorption α from (3.9), we obtain the expression (5.2) for the coefficient of electro-absorption. The analytical expressions for the functions $\Lambda_\ell^{u,g}(\omega)$ can be found in the asymptotic regions of the spectrum. For frequencies considerably lower than the energy of the edge, we have

$$\mathcal{E} = \ell\hbar\omega - \mathcal{E}_g - (b_e + b_h) - (\Delta_e + \Delta_h)\beta^2/8 < 0 \qquad |\mathcal{E}| \gg \mathcal{E}_0 \tag{6.2}$$

with

$$\Lambda_\ell^{u,g}(\omega) \sim (|\mathcal{E}|/\mathcal{E}_0)^{-3/4} e^{-4/3(|\mathcal{E}|/\mathcal{E}_0)^{3/2}}. \tag{6.3}$$

For considerably higher frequencies, that is when the edge $\mathcal{E} > 0$ and $\mathcal{E} \gg \mathcal{E}_0$, there is no effect of the uniform electric field. If these frequencies are higher, the edge of the miniband is lowered if $(\Delta_e + \Delta_h) > \mathcal{E}_0$ as $\Lambda_\ell^{u,g}(\omega)$ is approximately constant. In the bulk case, the absorption increases with increasing frequency.

Equation (6.3) describes the Franz–Keldysh effect [17] in the superlattice in the presence of a transverse uniform electric field. The electric field induces an absorption at an energy lower than the edge. This absorption decreases exponentially in the direction of decreasing energy. Thus the absorption edge moves to a lower energy by an amount $\mathcal{E}_0$. The full shift $\Delta\omega$ of the edge will depend also upon the oscillating electric field F_0 so that

$$\ell\hbar\Delta\omega \simeq (\Delta_e + \Delta_h)\beta^2/8 - \mathcal{E}_0.$$

In contrast to the bulk case, this shift depends not only upon the electric fields F_0 and E but also upon the total width of the electron and hole minibands $(\Delta_e + \Delta_h)$. A very recent set of experiments by Schmeller *et al* [18] has found strong subband absorption containing oscillations from an InGaAs/GaAs quantum well. The authors suppose that they are observing the Franz–Keldysh effect although no detailed theory was given. It is possible that our calculations may have some applicability to this case but this application has not been attempted.

7. Conclusions

The results obtained above can be used for the qualitative and quantitative analysis of the multiphoton magneto- and electro-absorption experimental spectra of semiconductor superlattices. They extend previous calculations by including more than one photon and thus provide additional information for the modelling of experimental data on real systems. The analysis emphasizes the importance of including time dependence in the calculation of absorption spectra of superlattice systems when the oscillating field is intense. It is important to distinguish between the various components of an observed absorption spectrum. The multiphoton contribution has been shown to be important in the presence of an intense optical wave. Another important contribution is that arising from excitonic transitions. Work on this is in progress and will form the subject of a later paper [19]. It would be interesting to compare our calculations with experiment.

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